

Name of College - S.S. College, J-Bad

Page-1

Dept - Mathematics

Topic - Problem based on Hyperbolic Function

Date - 07-04-2021

Time - 10 A.M to 11.30 A.M

B.Sc I (HONS)

By - Samrendrakumar

Problem $\rightarrow$ Separate into Real and Imaginary parts $\rightarrow$

(i) $\sin(x+iy)$ (ii) $\sinh(x+iy)$

Solution $\sin(x+iy) = \sin x \cos iy + \cos x \sin iy$
 $= \sin x \cosh y + i \cos x \sinh y$
 $= a + ib$

Re. $\sin(x+iy) = \sin x \cosh y$

Im $\sin(x+iy) = \cos x \sinh y$

(ii) $\sinh(x+iy) = -i \sin i(x+iy)$
 $= -i \sin(ix-y)$

$\sinh x = -i \sin ix$

$i^2 = -1$

$= -i [\sin ix \cos y - \cos ix \sin y]$
 $= -i \sin ix \cos y + i \cos ix \sin y$
 $= \sinh x \cos y + i \cosh x \sin y$

$\sinh x = -i \sin ix$
 $\cosh x = \cos ix$

$\therefore$ Real part of $\sinh(x+iy) = \sinh x \cos y$

Imaginary part of $\sinh(x+iy) = \cosh x \sin y$

Separate $\log \sin(\theta + i\phi)$ into real and imaginary parts —

$$\begin{aligned} \text{Solution: } \rightarrow \sin(\theta + i\phi) &= \sin\theta \cosh\phi + i \cos\theta \sinh\phi \\ &= \sin\theta \cosh\phi + i \cos\theta \sinh\phi \\ &= \alpha + i\beta \end{aligned}$$

Where $\alpha = \sin\theta \cosh\phi$

$\beta = \cos\theta \sinh\phi$

$$\begin{aligned} \text{Therefore, } \log \sin(\theta + i\phi) &= \log(\alpha + i\beta) \\ &= \frac{1}{2} \log(\alpha^2 + \beta^2) + i \tan^{-1} \frac{\beta}{\alpha} \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \text{Now } \alpha^2 + \beta^2 &= \sin^2\theta \cosh^2\phi + \cos^2\theta \sinh^2\phi \\ &= \sin^2\theta (1 + \sinh^2\phi) + (1 - \sin^2\theta) \sinh^2\phi \\ &= \sin^2\theta + \sinh^2\phi \end{aligned}$$

$$\frac{\beta}{\alpha} = \frac{\cos\theta \sinh\phi}{\sin\theta \cosh\phi} = \cot\theta \tanh\phi$$

Putting these values in (1) $\left\{ \begin{aligned} \log(m+in) &= \frac{1}{2} \log(m^2+n^2) + i \tan^{-1} \frac{n}{m} \end{aligned} \right.$

$$\begin{aligned} \log \sin(\theta + i\phi) &= \frac{1}{2} \log(\sin^2\theta + \sinh^2\phi) \\ &\quad + i \tan^{-1}(\cot\theta \tanh\phi) \end{aligned}$$

Problem If $\sin(\theta + i\phi) = x + iy$,

Prove that

$$\textcircled{I} \quad \frac{x^2}{\sin^2\theta} - \frac{y^2}{\cosh^2\phi} = 1$$

$$\textcircled{II} \quad \frac{x^2}{\cosh^2\phi} + \frac{y^2}{\sinh^2\phi} = 1$$

By the question

Page 3

$$\begin{aligned}x + iy &= \operatorname{Sei}(\theta + iq) \\&= \operatorname{Sei} \theta \operatorname{csig} + i \operatorname{cso} \operatorname{seiz} q \\&= \operatorname{Sei} \theta \operatorname{csh} q + i \operatorname{cso} \operatorname{senh} q.\end{aligned}$$

Equating real and imaginary parts

$$x = \operatorname{Sei} \theta \operatorname{csh} q \quad y = \operatorname{cso} \operatorname{senh} q.$$

$$\Rightarrow \operatorname{csh} q = \frac{x}{\operatorname{Sei} \theta} \quad \operatorname{senh} q = \frac{y}{\operatorname{cso}}$$

$$\text{Since } \operatorname{csh}^2 q - \operatorname{senh}^2 q = 1$$

$$\Rightarrow \frac{x^2}{\operatorname{Sei}^2 \theta} - \frac{y^2}{\operatorname{cso}^2 \theta} = 1$$

$$(ii) \quad \operatorname{Sei} \theta = \frac{x}{\operatorname{csh} q} \quad \operatorname{cso} \theta = \frac{y}{\operatorname{senh} q}$$

$$\therefore \operatorname{Sei}^2 \theta + \operatorname{cso}^2 \theta = 1$$

$$\Rightarrow \frac{x^2}{\operatorname{csh}^2 q} + \frac{y^2}{\operatorname{senh}^2 q} = 1$$

Problem 3. If $\operatorname{Sei}(\theta + iq) = u + iv$

Prove that $\operatorname{Sei}^2 \theta$ and $\operatorname{csh}^2 q$ are the roots of the Equation

$$t^2 - t(1 + u^2 + v^2) + u^2 = 0.$$

$$\text{Soln. } \operatorname{Sei}(\theta + iq) = u + iv$$

$$\Rightarrow \operatorname{Sei} \theta \operatorname{csig} + i \operatorname{cso} \operatorname{seiz} q = u + iv$$

$$\Rightarrow \operatorname{Sei} \theta \operatorname{csh} q + i \operatorname{cso} \operatorname{senh} q = u + iv$$

Equating real parts.

Page-4

$$U = \sin \theta \cosh \phi$$

$$V = \cos \theta \sinh \phi$$

$$\begin{aligned} U^2 + V^2 &= \sin^2 \theta \cosh^2 \phi + \cos^2 \theta \sinh^2 \phi \\ &= \sin^2 \theta \cosh^2 \phi + (1 - \sin^2 \theta) \sinh^2 \phi \\ &= \sin^2 \theta \cosh^2 \phi + \sinh^2 \phi - (1 - \sin^2 \theta)(\cosh^2 \phi - 1) \\ &= \sin^2 \theta \cosh^2 \phi + \cosh^2 \phi - 1 - \sin^2 \theta \cosh^2 \phi + \sin^2 \theta \\ &= \sin^2 \theta + \cosh^2 \phi - 1 \end{aligned}$$

$$\begin{aligned} \therefore \text{Sum of the roots} &= \sin^2 \theta + \cosh^2 \phi = 1 + U^2 + V^2 \\ \text{And product of the roots} &= \sin^2 \theta \cosh^2 \phi = U^2 \end{aligned}$$

$\therefore$ quadratic Equation whose roots are $\sin^2 \theta$ and $\cosh^2 \phi$ is

$$\begin{aligned} t^2 - [\sin^2 \theta + \cosh^2 \phi]t + \sin^2 \theta \cosh^2 \phi &= 0 \\ \Rightarrow t^2 - (1 + U^2 + V^2)t + U^2 &= 0 \end{aligned}$$

$$\xrightarrow{\quad 0 \quad} \xrightarrow{\quad 0 \quad} \xrightarrow{\quad \quad}$$