Name of College - S.S College, J-Bad

Deptt - Mathematics

Topic - Problem based on Hyperbolic Function

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B.Sc I (HONS)

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Problem :→ Separate into Real and imaginary parts :→

(i) $\sin(x+iy)$ (ii) $\sinh(x+iy)$

Solution $\sin(x+iy) = \sin x \cos iy + \cos x \sin iy$

$= \sin x \cosh y + i \cos x \sinh y$

$= a + ib$

$Re. \sin(x+iy) = \sin x \cosh y$

$Im \sin(x+iy) = \cos x \sinh y$

$\sinh x = -i \sin ix$

$i^2 = -1$

(ii) $\sinh(x+iy) = -i \sin i(x+iy)$

$= -i \sin(ix - y)$

$= -i [\sin ix \cos y - \cos ix \sin y]$

$= -i \sin ix \cos y + i \cos ix \sin y$

$\sinh x = -i \sin ix$
$\cosh x = \cos ix$

$= \sinh x \cos y + i \cosh x \sin y$

$\therefore$ Real part of $\sinh(x+iy) = \sinh x \cos y$

Imaginary part of $\sinh(x+iy) = \cosh x \sin y$

Separate $\log \sin(\theta + i\varphi)$ into real and imaginary parts —

Solution :→ $\sin(\theta + i\varphi) = \sin\theta \cos i\varphi + \cos\theta \sin i\varphi$

$= \sin\theta \cosh\varphi + i\cos\theta \sinh\varphi$

$= \alpha + i\beta$.

Where $\alpha = \sin\theta \cosh\varphi$

$\beta = \cos\theta \sinh\varphi$.

Therefore, $\log \sin(\theta + i\varphi) = \log(\alpha + i\beta)$

$= \frac{1}{2}\log(\alpha^2 + \beta^2) + i\tan^{-1}\frac{\beta}{\alpha}$ —— (1)

Now $\alpha^2 + \beta^2 = \sin^2\theta \cosh^2\varphi + \cos^2\theta \sinh^2\varphi$

$= \sin^2\theta\,[1 + \sinh^2\varphi] + (1 - \sin^2\theta)\sinh^2\varphi$

$= \sin^2\theta + \sinh^2\varphi$.

$$\frac{\beta}{\alpha} = \frac{\cos\theta \sinh\varphi}{\sin\theta \cosh\varphi} = \cot\theta \tanh\varphi$$

Putting these values in (1)

$\left[\log(m + in) = \frac{1}{2}\log(m^2 + n^2) + i\tan^{-1}\frac{n}{m} \right]$

$$\log \sin(\theta + i\varphi) = \frac{1}{2}\log\left[\sin^2\theta + \sinh^2\varphi\right] + i\tan^{-1}(\cot\theta \tanh\varphi)$$

Problem If $\sin(\theta + i\varphi) = x + iy$,

Prove that

(i) $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$ (ii) $\frac{x^2}{\cosh^2\varphi} + \frac{y^2}{\sinh^2\varphi} = 1$

By the question

$$x+iy = \sin(\theta+i\varphi)$$
$$= \sin\theta\cos i\varphi + \cos\theta\sin i\varphi$$
$$= \sin\theta\cosh\varphi + i\cos\theta\sinh\varphi.$$

Equating real and imaginary parts

$$x = \sin\theta\cosh\varphi \qquad y = \cos\theta\sinh\varphi.$$

$$\Rightarrow \cosh\varphi = \frac{x}{\sin\theta} \qquad \sinh\varphi = \frac{y}{\cos\theta}$$

Since $\cosh^2\varphi - \sinh^2\varphi = 1$

$$\Rightarrow \frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

(ii) $\sin\theta = \dfrac{x}{\cosh\varphi} \qquad \cos\theta = \dfrac{y}{\sinh\varphi}$

$$\because \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \frac{x^2}{\cosh^2\varphi} + \frac{y^2}{\sinh^2\varphi} = 1$$

Problem 3. If $\sin(\theta+i\varphi) = u+iv$

Prove that $\sin^2\theta$ and $\cosh^2\varphi$ are the roots of the Equation

$$t^2 - t(1+u^2+v^2) + u^2 = 0.$$

Since $\sin(\theta+i\varphi) = u+iv$.

$$\Rightarrow \sin\theta\cos i\varphi + \cos\theta\sin i\varphi = u+iv.$$
$$\Rightarrow \sin\theta\cosh\varphi + i\cos\theta\sinh\varphi = u+iv.$$

Equating real parts.

$$u = \sin\theta \cosh\varphi$$
$$v = \cos\theta \sinh\varphi$$

$$u^2+v^2 = \sin^2\theta \cosh^2\varphi + \cos^2\theta \sinh^2\varphi$$
$$= \sin^2\theta \cosh^2\varphi + (1-\sin^2\theta)\sinh^2\varphi$$
$$= \sin^2\theta \cosh^2\varphi + \cancel{\sinh^2\varphi} - (1-\sin^2\theta)(\cosh^2\varphi-1)$$
$$= \cancel{\sin^2\theta \cosh^2\varphi} + \cosh^2\varphi - 1 - \cancel{\sin^2\theta \cosh^2\varphi} + \sin^2\theta$$
$$= \sin^2\theta + \cosh^2\varphi - 1$$

$\therefore$ Sum of the roots $= \sin^2\theta + \cosh^2\varphi = 1+u^2+v^2$

and product of the roots

$$= \sin^2\theta \cdot \cosh^2\varphi = u^2$$

$\therefore$ quadratic equation whose roots are $\sin^2\theta$ and $\cosh^2\varphi$ is

$$t^2 - [\sin^2\theta + \cosh^2\varphi]t + \sin^2\theta \cdot \cosh^2\varphi = 0$$

$$\Rightarrow t^2 - (1+u^2+v^2)t + u^2 = 0$$